You are given 4 hours. Each problem is worth 7 points. Each question is worth 1 point.
Problems are meant to be kept confidential even after the test until 1 April 2024.
All results must be proven to obtain the 7 points.

Problem 1. *Which $n \times m$ boards can be tiled using L-shaped tetrominoes (that is, L-shaped pieces with 4 squares)?*

Problem 2. *A sequence of positive integers a_n begins with $a_1 = a$ and $a_2 = b$ for positive integers a and b. Subsequent terms in the sequence satisfy the following two rules for all positive integers n:*

$$a_{2n+1} = a_{2n}a_{2n-1}, a_{2n+2} = a_{2n+1} + 4.$$

Exactly m of the numbers $a_1, a_2, a_3, ..., a_2022$ are square numbers. What is the maximum possible value of m?

Problem 3. *Let ABC be a triangle with an obtuse angle A and incentre I. Circles ABI and ACI intersect BC again at X and Y respectively. The lines AX and BI meet at P, and the lines AY and CI meet at Q. Prove that $BCQP$ is cyclic.*

Problem 4. *Let x, y, z be positive real numbers. Prove that*

$$(xy^2 + yz^2 + zx^2)(x^2y + y^2z + z^2x)(xy + yz + zx)(x + y + z) \geq 1/9(xyz)^3.$$

Question 1. *Find the remainder mod 11 of the number of ways of turning 10 lights on or off.*

a)1, *b*)3, *c*)5, *d*)7, *e*)9

Question 2. *Find xyz given that*

$$3^x * 3^y * 3^z = 9$$
$$2^x + 2^y + 2^z = \frac{13}{2}$$
$$x + y = 0$$

a) -2, *b*) -3, *c*) $-\frac{1}{2}$, *d*) -4, *e*) -1

Question 3. *Iker has just taken out his NID card, because he need it to travel to Winchester. But David and Alexandro, two of his friends, have decided to steal it. But as they are a bit clumsy, they have only stolen 3 digit. Now the DNI is something like this: $1x94y6z2E$. Were x, y, z are non-negative integers, between 0 and 9. Now you have to help Iker to find them. Here are some clues. The number $1x94y6z2$ is a multiple of 12, but not of 8. z is bigger than y y is bigger than x It is also known that the letter in the DNI corresponds with the remainder of dividing $1x94y6z2$ by 23. As the letter is E, the remainder is equal to 22. Iker loves '1', and he remember that, unluckily, his NID doesn't have any 1 in his digits. Help Iker to know the value of $x + y + z$.*

a) 23 b) 20 c) 17 d) 26 e) 22

Question 4. *How many times does* 10 *divide* 310!*?*
a)75, *b*)76, *c*)77, *d*)78, *e*)79

Question 5. *Determine the number of real solutions* (x, y, z) *to the following system of equations*

$$x + y + z = 13$$

$$xy + xz + yz = 47.75$$

$$xyz = 35.75$$

a)0, *b*)2, *c*)3, *d*)4, *e*)6

Question 6. *Consider a necklace with* 8 *red beads and* 32 *green beads. We will say a necklace is 'based' if between any two red beads there are at least two green beads. How many based necklaces are there?*
a)0, *b*)245336, *c*)30667, *d*)10222, *e*)120534

Question 7. *Ana decides to play with the random number generator and generates* 4 *different random numbers from* 1 *to* 9*. What is the probability* 3 *of the* 4 *numbers she has generated add up to* 15*.*

Question 8. *Triangle* ABC *with* $AB = 14$*,* $AC = 30$*,* $BC = 40$ *is inscribed in a circle* ω*. The tangents to* ω *at* B *and* C *meet at a point* T*. The tangent to* ω *at* A *intersects the perpendicular bisector of* AT *at point* P*. Compute the area of triangle* PBC*.*

Question 9. *Let* $f(x) = x^2 + 6x + 1$ *and* R *be the set of all points* (x, y) *in the coordinate plane such that* $f(x) + f(y) \leq 0$ *and* $f(x) - f(y) \leq 0$*. The best approximation for the area of* R *is a*)21, *b*)22, *c*)23, *d*)24, *e*)25.

Question 10. *The number of squarefree integers from* $\{1, 2, \ldots, 1000\}$ *is closest to:*
a)200, *b*)300, *c*)400, *d*)500, *e*)600